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ABSTRACT

This is the second of four guidebooks on minimum content designed to strengthen fundamental concepts which are basic preparation for Algebra I. This booklet covers integers, rational and irrational numbers, real number properties and operations, graphing in one dimension, and open sentences. The overall course goals are stated, then, for each topic covered, a list of performance objectives, a course outline, references to state-adopted texts, and suggested teaching strategies is provided. Pretest and posttest items, plus a list of eight references are included. For the first guidebook in this set, see ED 062 181. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

PRE-ALGEBRA 2

5210.12

MATHEMATICS

DIVISION OF INSTRUCTION•1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

PRE-ALGEBRA 2

5210.12

(EXPERIMENTAL)

Written by

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for the

DIVISION OF INSTRUCTION
Dade County Public Schools
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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOGUE DESCRIPTION

The second of four quins designed to strengthen fundamental concepts which are basic preparation for Algebra 1. Includes an introduction to irrational numbers, real number properties and operations, graphing in one dimension and open sentences.

Designed for the student who has mastered the skills and concepts of Pre-Algebra 1.

TABLE OF CONTENTS

OVERALL GOALS

1. To introduce the set of integers, and develop skill in computation with integers.

2. To begin to develop an understanding of the real number system.

3. To introduce the properties of equality and develop skill in applying the properties to the solution of equations.

4. To graph solution sets of open sentences on the real number line.

5. To provide further work in the solutions of verbal problems.

KEY TO REFERENCES
(*State adopted)

K (2) - Keedy, Mervin; Jameson, Richard; and Johnson, Patricia. Exploring Modern Mathematics, Book II. New York: Holt, Rinehart and Winston, Inc., 1963.

Mc(8) - McSwain, E.T.; Brown, K.W.; Gundlach, B.H.; and Cooke, R.J. Mathematics 8. River Forest, Illinois: Laidlaw Brothers, 1965.

N - Nichols, Eugene D., Pre-Algebra Mathematics. New York: Holt, Rinehart and Winston, Inc., 1965.

*SW - Skeen, Kenneth C. and Whitmore, Edward H. Modern Math, Book I. Atlanta: L.W. Singer Co., 1966.

*SM - Sobel, Max A. and Maletsky, Evan M. Essentials of Mathematics, Skills and Concepts 3. Upper Montclair, New Jersey: Ginn and Company, 1969.

I. INTEGERS

PERFORMANCE OBJECTIVES

The student will:

1. Define the set of integers by rule or roster.
2. Perform the four basic operations using integers.

COURSE OUTLINE

I. Integers

- A. Definition
  - 1. By rule
  - 2. By roster
- B. Addition
  - 1. Using the number line
  - 2. Property of opposites
  - 3. Practice
- C. Subtraction
  - 1. Definition
  - 2. Practice
- D. Multiplication and Division
  - 1. Rules of signs
  - 2. Practice
- E. Computation with more than one operation.
- F. Verbal Problems

REFERENCES

K (2) -(pp. 6-31, 137-139) Good development of integers, their addition and subtraction. Little work with multiplication and none with division.

Mc(8) -(pp. 46-57) Covers the operations with integers and also the properties of the operations.

N -(pp. 224-273) Presents the operations with directed numbers hence exercises include all rational numbers not only integers. Properties are also included.

SW -(pp. 73-92, 100-102, 241-248) Integers are introduced as the extension of the set of whole numbers and the number line is used to develop the rules of addition and subtraction. Later in the book, multiplication and division are presented.

SM -(pp. 104-132) Integers are introduced through work with the number line and addition is introduced with a monograph. The rules of multiplication and division are developed by completing a multiplication table.

SUGGESTED STRATEGIES

1. The number line is a good device to use in introducing integers and the addition of integers. Be sure that students develop the next step and learn to add without the aid of the number line.

2. Students should be able to add integers efficiently before being introduced to subtraction. Subtraction should be defined as the "addition of the inverse" and the student should rewrite each subtraction problem as an addition problem. Trying to show subtraction on the number line is likely to provide more confusion than help.

3. Absolute value is an important concept and should be stressed. Several definitions of absolute value should be discussed to aid the student in his understanding of this concept.

4. Be sure to work with the number line in comparing integers so the student sees that $-6 < -2$ etc.

5. Multiplication of integers can be introduced by developing patterns as in Nichols' Pre-Algebra, page 222.

II. REAL NUMBERS

PERFORMANCE OBJECTIVES

The student will:

1. Express any rational number in the decimal form.
2. Identify real numbers as rational or irrational
3. Match real numbers with their graphs on the real number line.

COURSE OUTLINE

II. Real Numbers
- A. Rational numbers
  1. Definition as ratio of integers.
  2. Definition as repeating or terminating decimals.
  3. Expressing rationals in equivalent forms.
- B. Irrational numbers
  1. Definition as non-repeating, non-terminating decimals.
  2. Other forms
     a. roots
     b. π
- C. Real numbers
  1. Definition as the union of rational and irrational numbers.
  2. Locating real numbers on the number line.

REFERENCES

K (2) - (pp. 164-167, 198) Nothing on irrational or real numbers and little on the set of rationals, although it does go into computation with rationals.

Mc (8) - (pp. 119, 120, 133, 142, 143) Introduces the set of rationals, but then proceeds to work with non-negative rationals.

N - (pp. 126-152, 192-209) Good development of the relationship between the set of real numbers and its subsets. Deals only with positive real numbers calling them the "real numbers of arithmetic." Interesting treatment on the development of $\sqrt{2}$.

SW - (pp. 116-121, 248-255) Develops the set of rational numbers, but has little on irrationals and reals.

SM - (pp. 448, 449) Refers to real numbers only as the universal set in graphing. No work with irrationals.

SUGGESTED STRATEGIES

1. The purpose of this section is to introduce the student to the set of irrational numbers and the set of real numbers and to develop the relationships that exist between these sets and the set of rational numbers.

2. Expressing rational numbers in their decimal form provides good practice in division.

3. It would be good to review all the sets of numbers with which the student has worked. A tree diagram or other chart is useful in presenting the relationship between the sets and can make a colorful bulletin board display.

4. Another device for helping the students learn the elements in the different sets is to give the students a number and have them list all the sets to which it belongs. Be sure to stress zero, for it often is a stumbling block for students.

III. OPEN SENTENCES

PERFORMANCE OBJECTIVES

The student will:

1. Solve equations with integral coefficients using the addition property.
2. Solve equations with integral coefficients using the multiplication property.
3. Solve equations with integral coefficients using both addition and multiplication properties and the distributive property in combining like terms.
4. Solve simple inequalities.

COURSE OUTLINE

III. Open Sentences

- A. Definitions
  1. Variable
  2. Open sentence
  3. Equations
- B. Inverse operations
  1. Addition-Subtraction
  2. Multiplication-Division
- C. Solution
  1. $x \div a = b$
  2. $a \times = b$
- D. Distributive property
  1. Definition
  2. Use in simplifying number phrases
  3. Use in combining similar terms
- E. Solutions
  1. $a x + b x + c = d$
  2. $a x + b = c x + d$
- F. Inequalities

REFERENCES

K (2) - (pp 285-309) Assumes the student is familiar with open sentences from earlier work. Covers solutions of the different types of equations, but is not restricted to equations with integral coefficients.

Mc (8) - (pp 72-79) Introduces the division property as well as the addition and multiplication properties of equality. The subtraction property is included with addition. A complete solution of $3t + 8 = 59$ with all steps and reasons is given on page 78.

N - (pp 251-253) Uses the cover up method to solve equations. Does not go into the properties of equations.

SW - (pp 92-96, 161-169, 264-272) Equations are compared to a balance. Properties for all four operations are introduced. Combination of properties introduced with rationals.

SM - (pp 358-376) Detailed development of the properties with much practice. Uses a flow chart and its reverse to explain the properties and how to apply them.

SUGGESTED STRATEGIES

1. Have students write all steps in the solutions of equations and indicate the property used in each step.

   Example: $2x + 3 = 7$

   A (-3) $2x = 4$

   M $(\frac{1}{2})$ $x = 2$

2. Encourage students to check problems by using the substitution principle.

3. The flow chart method in Essentials of Mathematics 3 is helpful if the students are familiar with flow charts.

4. The cover up method for solving equations in Nichols' Pre-Algebra, page 251, is a good introduction to solving equations, but the properties of equality should be introduced and used also.

5. Although most of the references do not cover the use of the distributive property in combining similar terms and solving equations such as $a x + b x + c = d$ it should be covered. The distributive property is so important in future mathematical work that it should be reviewed and used whenever possible.

6. Confusion will be avoided when solving inequalities if, in an inequality such as $5 - 2x > 17$, the student changes the inequality to $5 > 17 + 2x$ so the coefficient of x is positive. If this is always done, then there is no need to reverse the inequality sign.

IV. GRAPHING ON THE REAL NUMBER LINE

PERFORMANCE OBJECTIVES

The student will:

1. Graph the solution sets of equations.
2. Identify geometric sets that result when graphing inequalities on the real number line.
3. Graph inequalities on the real number line for specified replacement sets.

COURSE OUTLINE

IV. Graphing on the real number line.

- A. The real number line
  1. Drawing the line
  2. Locating points
  3. One-to-one correspondence between real numbers and the points on a line.
- B. Geometric sets
  1. Identifying points, lines, rays, segments, open segments, half lines, half open segments.
  2. Labeling the above sets.
- C. Graphing
  1. Equations
  2. Inequalities
     - a. Integers as the replacement set.
     - b. Reals as the replacement set.

REFERENCES

K (2) - Graphing on a line is not covered.

Mc (8) - (pp 149-150, 302-303) Incomplete coverage of geometric sets. Graphing on a line reviewed as an introduction to graphing on a plane.

N - (pp 255-259, 321-325) Has graphing exercises using the set of integers and set of real numbers as the replacement set.

SW - (pp. 50-54, 97-100, 272-277) Shows graphing inequalities with integers then with reals. Does not cover the geometric sets which result from the graphing.

SM - (pp 262-268, 396-416) Good coverage of the topics, with sufficient practice.

SUGGESTED STRATEGIES

1. Use the number line to illustrate the property of density and also to illustrate the transitive property.

2. Stress that the graph of an inequality depends on the replacement set. Have students graph the same inequality using whole numbers, integers and real numbers as the replacement set.

SAMPLE PRETEST ITEMS

1. Give an illustration of each of the following properties of rational numbers:

   a. Commutative property for addition

   b. Associative property for multiplication

   c. The identity property for addition

   d. The identity property for multiplication

2. In each case, identify, by complete name or accepted abbreviation, the property illustrated:

   a. $a \cdot 1 = a$

   b $(a + b) + c = a + (b + c)$

   c. $\frac{1}{2} + \frac{2}{3} = \frac{2}{3} + \frac{1}{2}$

   d. $9 + 0 = 9$

3. Perform the operations as indicated. All results must be reduced to lowest terms.

   a. $\frac{10}{20} + \frac{2}{20} =$

   b. $1\frac{1}{2} - 1\frac{1}{4} =$

   c. $4.2 - 1.3 =$

   d. $100 - .75 =$

   e. $\frac{3}{7} - \frac{2}{21} =$

   f. $\frac{3}{4} \times \frac{5}{5} =$

   g. $6 - \frac{3}{4} =$

   h. $\frac{1}{2} \times \frac{1}{2} =$

   i. $29 \times .01 =$

   j. $4.2 \times .06 =$

   k. $\frac{2}{3} \div \frac{4}{5} =$

   l. $6.4 \div .2 =$

   m. $\frac{4}{5} + \frac{2}{3} =$

   n. $\frac{1}{10} \div 3 =$

   o. $\frac{5}{12} + \frac{1}{4} =$

   p. $1\frac{1}{8} - \frac{1}{2} =$

   q. $6.2 - .043$

4. Identify whether each of the following is true or false by circling T or F:

T F a. $\frac{6}{3} < \frac{16}{4}$ T F c. $\frac{3}{4} > \frac{74}{100}$

T F b. $\frac{15}{3} = \frac{20}{5}$ T F d. $1.407 > 2.1$

5. Using rules of divisibility, circle every number on the right which is a divisor of the given natural number.

a. 372 2,3,4,5,6,8,9, 10

b. 3670 2,3,4,5,6,8,9, 10

c. 495 2,3,4,5,6,8,9, 10

d. 840 2,3,4,5,6,8,9, 10

6. Circle each prime number in the set below:

1, 5, 7, 8, 9, 11, 17, 18, 21, 27, 87, 92, 225

7. Give the prime factorization for each:

a. 108 b. 1540

8. Find the greatest common factor for each pair of natural numbers:

a. 48, 120 b. 105, 150

9. Determine the least common multiple for each pair of natural numbers:

a. 15, 25 b. 36, 48

10. Using numerals and variables express the following as number phrases or sentences:

a. Some number plus six equals fourteen.

b. Eight less than a number.

c. Three times a number is thirty.

d. Two more than twice a number.

KEY TO PRETEST

1. Examples of possible answers.

   a. $3 + 5 = 5 + 3$

   b. $(2 \times 3) \times 4 = 2 \times (3 \times 4)$

   c. $5 + 0 = 5$

   d. $6 \times 1 = 6$

2. a. Identity property for multiplication

   b. Associative property for addition

   c. Commutative property for addition

   d. Identity property for addition

3. a. $\frac{3}{5}$

   b. $\frac{1}{4}$

   c. 2.9

   d. 99.25

   e. $\frac{7}{21}$

   f. $\frac{3}{4}$

   g. $5\frac{1}{4}$

   h. $\frac{1}{4}$

   i. .29

   j. .252

   k. $\frac{5}{6}$

   l. 32

   m. $1\frac{7}{15}$

   n. $\frac{1}{30}$

   o. $\frac{2}{3}$

   p. $\frac{5}{8}$

   q. 6.157

4. a. T b. F c. T d. F

5. a. 2, 3, 4, 6

   b. 2, 5, 10

   c. 3, 5, 9

   d. 2, 3, 4, 5, 6, 8, 10

6. 5, 7, 11, 17

7. a. $2^2 \cdot 3^3$ b. $2^2 \cdot 5 \cdot 7 \cdot 11$

8. a. 24 b. 15

9. a. 75 b. 144

10. a. $n + 6 = 14$

b. $n - 8$

c. $3n = 30$

d. $2n + 2$

SAMPLE POSTTEST ITEMS

I. 1. Define the set of integers

2. Perform the indicated operations

a. $5 - (-3)$

b. $(-4) \cdot (-7)$

c. $-17 + 4$

d. $-15 \div 3$

e. $-21 - 13$

f. $6 - 21$

g. $16 \cdot (-4)$

h. $-9 + (-12)$

i. $-26 - (-14)$

j. $-45 \div (-15)$

II. 1. Change to decimal form.

a. $\frac{1}{5}$ b. $\frac{22}{4}$ c. $\frac{2}{7}$ d. $\frac{5}{8}$

2. Which of the following represent rational numbers?

a. $\frac{1}{2}$ b. -3 c. $\sqrt{2}$ d. 0

e. 1.3 f. $.6\overline{6}$ g. .161661666...

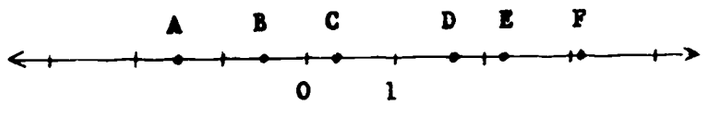
3. Which of the following represent irrational numbers?

a. $.83\overline{7}$ b. π c. $\sqrt{4}$ d. 0

e. .o1001001 f. $\sqrt{10}$ g. -4.3

4. Which lettered point on the number line is the graph of each of the points below?

a. 3 b. π c. $-\frac{3}{2}$ d. .3



III. Write a complete solution for each of the following equations:

1. a. $x - 3 = 15$ b. $14 + a = 29$

2. a. $5x = 45$ b. $\frac{c}{3} = 19$ c. $\frac{4y}{7} = 36$

3. a. $2x + 3 = 13$ b. $16 - 3x = -11$

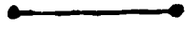
c. $\frac{d}{4} - 2 = 2$ d. $5a + 2a - 7 = 42$

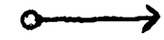
e. $3 - 57 + 4 + 2y = 46$ f. $4x + 7 = x - 8$

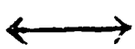
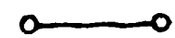
IV. 1. Graph each of the following on a number line:

a. (3, 5) b. (-7 0, 3)

2. Name each of sets pictured below:

a.  d.  g. 

b. e. 

c.  f. 

3. Graph the solution set of each inequality

a. $x < 3, \ x \in R$ d. $-1 \leq c < 5, \ C \in R$

b. $2 < n < 5, \ n \in W$

c. $t \geq -2, \ t \in I$

KEY TO POSTTEST

I. 1. The set of integers consists of zero, the natural numbers and an additive inverse for each natural number.

2. a. 8 f. -15

b. 28 g. -64

c. -13 h. -21

d. -5 i. -12

e. -34 j. 3

II. 1. a. .2 b. 5.5 c. $.\overline{285714}$ d. .625

2. a. $\frac{1}{2}$, -3, 0, 1.3, $6.\overline{6}$

b. π , $\sqrt{10}$

3. a. D b. F c. A d. C

III. 1. a. $x-3 = 15$

$x = 15 + 3$

$x = 18$

b. $14 + a = 29$

$a = 29 - 14$

$a = 15$

2. a. $5x = 45$

$x = \frac{45}{5}$

$x = 9$

b. $\frac{c}{3} = 19$

$c = 19 \times 3$

$c = 57$

c. $\frac{4y}{7} = 36$

$y = 36 \times \frac{7}{4}$

$y = 63$

3. a. $2x + 3 = 13$

$2x = 13 - 3$

$x = \frac{10}{2}$

$x = 5$

b. $16 - 3x = -11$

$-3x = -11 - 16$

$x = \frac{-27}{-3}$

$x = 9$

c. $\frac{d}{4} - 2 = 2$

$\frac{d}{4} = 2 + 2$

$d = 4 \times 4$

$d = 16$

d. $5a + 2a - 7 = 42$

$7a - 7 = 42$

$7a = \frac{49}{7}$

$a = 7$

e. $3 - 5y + 4 + 2y = 46$

$7 - 3y = 46$

$-3y = 46 - 7$

$y = \frac{39}{-3}$

$y = -13$

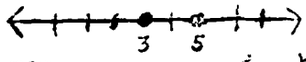
f. $4x + 7 = x - 8$

$4x - x = -8 - 7$

$3x = -15$

$x = -5$

IV. 1. a.



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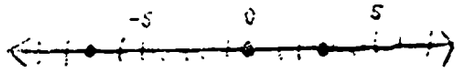


2. a. ray

b. point

c. line

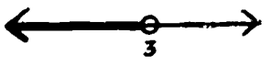
d. half-open segment

e. half line

f. open segment

g. segment

3. a.



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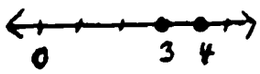

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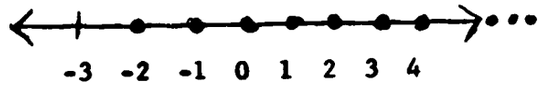

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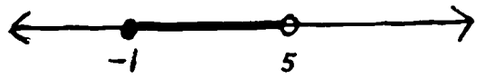

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